

How to Make Cubes and Fundamental Particles

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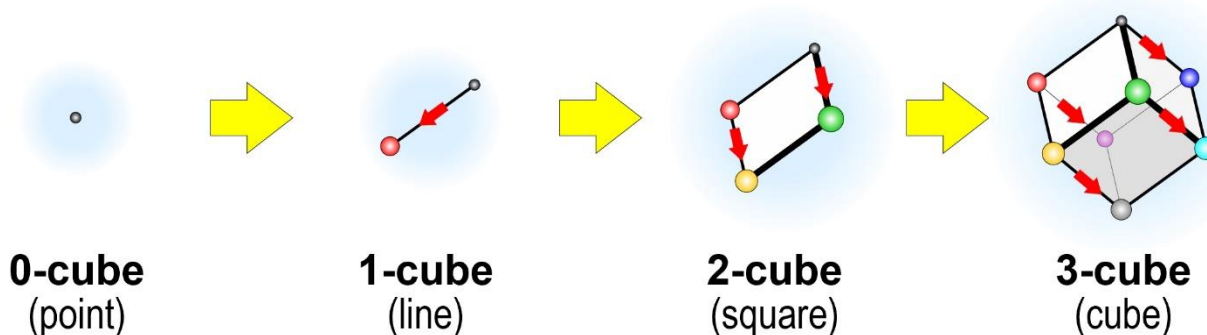


Figure 1. *Constructing the first four cubes.*

Cubes are fun! Did you know that there are an infinite number of them and that new ones are easy to build?

The lead figure, [Figure 1](#), shows the first four cubes in this infinite sequence. The last one, the 3-cube, is what most folks think of when they hear “cube.”

Here’s the question to ask, however: If you imagine being in a space with fewer than three dimensions, what shapes most resemble cubes — enough so that one can justify calling them the “cubes” of that number of dimensions? The figure shows the most cube-like shapes in spaces with less than three dimensions and their more common names. The first is the point. The point is the only cube possible if you have *no* dimensions — no number lines — available, so folks put a zero in front of it (the *0-cube*) to indicate the number of dimensions in which this cube resides.

Next is the line — or, more precisely, a fixed-length *line segment*. A line segment is the nearest thing to a cube with only one dimension and thus is also called a *1-cube*.

Next up is the square, and finally, the shape is beginning to look like what we normally think of as a cube. This version of a cube resides on a flat sheet with two dimensions, and is also called a *2-cube*.

Then, at last, comes the *real* cube, the one we automatically think of when we hear the word cube. Since space has three dimensions — left-right, up-down, and forward-backward — an ordinary cube is also called a *3-cube*.

[Figure 1](#) also displays the promise method for building a new, higher-dimensional cube from whatever one you have in hand. You *copy* the cube you have and push the copy in some new direction. For the first three new cubes, the red arrows in the figure show the new directions for pushing the copy of the cube. Draw lines to show the paths over which you dragged the corners of the cube copy, and *voila!* — you have a new cube with one more dimension.

I’ve drawn the corners of these cubes as differently colored spheres to make them stand out and help distinguish them from each other. All but the smaller first or “zero point” spheres are supposed to be the same size, with perspective making the closer spheres look a bit bigger. I’ve balanced the final 3-cube on one of its corners to make what comes next easier: Constructing more cubes that don’t fit into three-dimensional space but, surprisingly, are still easy to draw using the copy-and-push method.



Figure 2 shows how the copy-and-push method easily creates two more versions of cubes that, in their full glory, exist only in four-dimensional or five-dimensional space. The trick is simple: Since we don't have direct access to more than three dimensions of space, we "borrow" arbitrary directions and declare that to be the stand-in for the higher dimension. Somewhat remarkably, the resulting drawings show exactly the edges one would see if the cubes were embedded in four- or five-dimensional space, making it easy to find important connection relationships in the higher cubes.

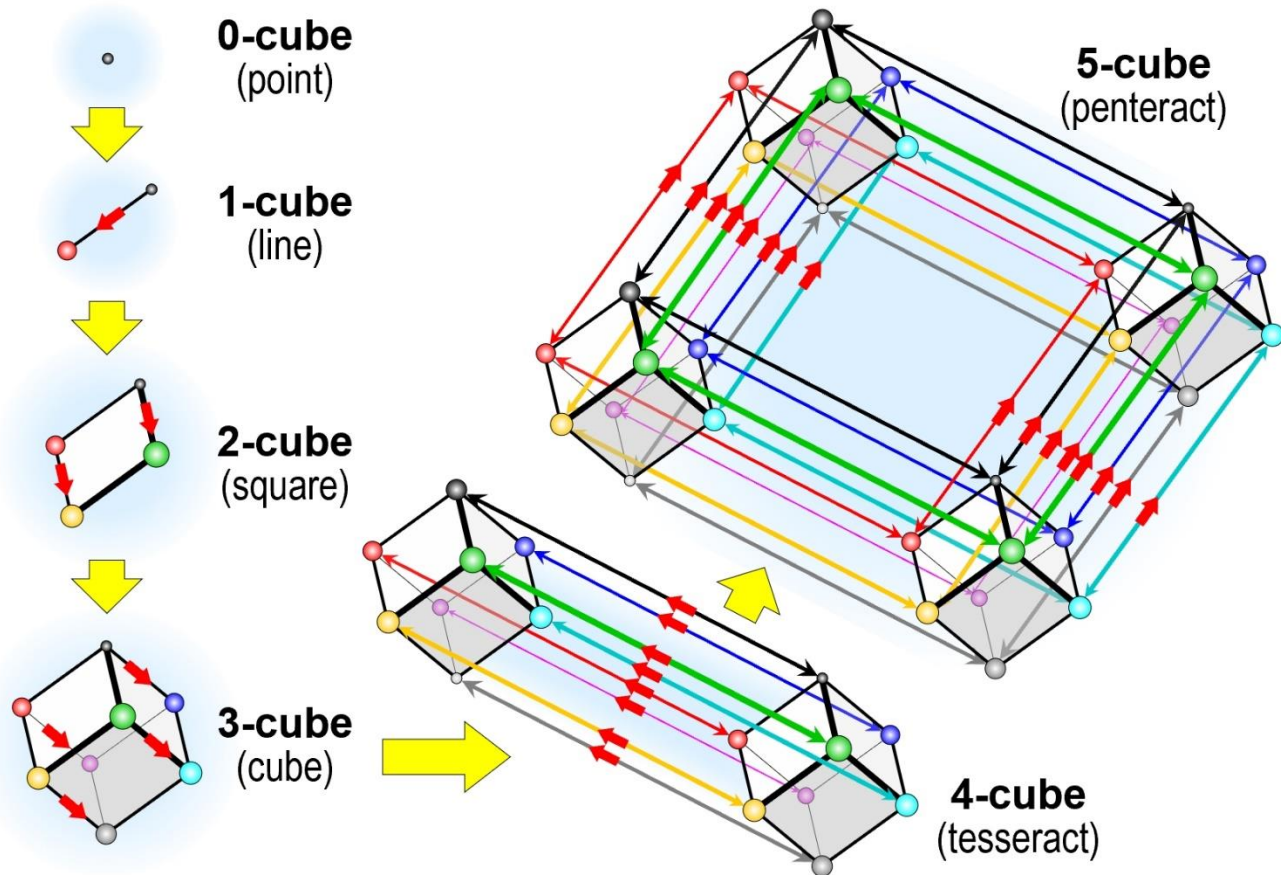


Figure 2. Constructing the first six cubes (tesseracts and penteracts).

These two higher-dimensional cubes have special names. Many call the 4-cube a *tesseract*, a term that often shows up in science fiction. The name of the 5-cube is much less common but makes sense: the *penteract*, which merges “penta” for five dimensions with the “act” of the tesseract.

There are many other ways to create and describe higher-dimensional cubes, but most of them lack the simplicity of saying, “two copies pushed apart in some new direction.” Getting one’s head around such a simple procedure is easy, and the result is surprisingly easy to draw.

Sheldon’s Cube

One of the more subtle jokes — more accurately, a pointed dig — in the popular TV series *The Big Bang Theory* was its use of an uncommon first name for the main character, Sheldon Cooper, a theoretical physicist fascinated by something called supersymmetry. In real life, Sheldon Glashow, who mocked supersymmetry, was one of the founders of the Standard Model of particle physics. This remarkable work transformed the chaotic results of particle colliders into a complicated, sometimes arbitrary, but highly predictive comprehensive theory that matched beautifully with observations. It is as much a foundation of physics as relativity and quantum theory.



Sheldon Glashow also found cubes interesting. In a speech celebrating the then-new Standard Model, Glashow tossed in, almost as an aside, the observation that particles that make up solid matter — which physicists call *fermions* — form a curiously simple cube structure. Figure 3 shows Glashow's original drawing [1] of this cube.

The Glashow Fermion Cube



S. L. Glashow, "The Future of Elementary Particle Physics [HUTP-79/A059]," *Harvard University Preprints*, Jul. 1979. Available: <https://inspirehep.net/literature/144466>. Page 29, Section III, *Let the Desert Bloom!* S. L. Glashow's original hand drawing.

Figure 3. The Glashow 3-cube shows all possible "matter particles" (fermions).

Figure 4 provides a cleaner, easier-to-read version of Glashow's cube with slightly updated notation:

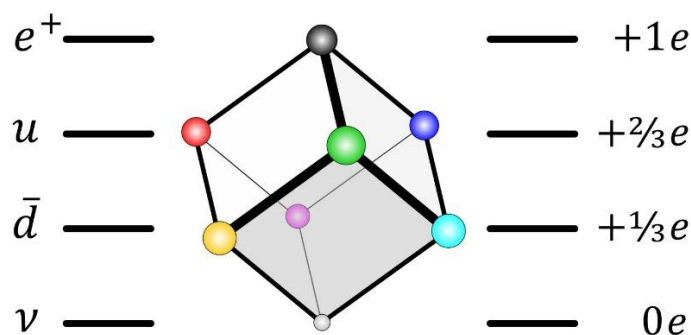


Figure 4. A cleaner version of the Glashow 3-cube.

Glashow balanced his cube on one corner and defined the vertical diagonal of the cube to be an ordinary electric charge, though in a strangely fractured form. The bottom corner of the cube has no electric charge, the top corner has the same charge as a proton, and the two layers in between have $\frac{1}{3}$ and $\frac{2}{3}$ fractions of positive charge. However, the positively charged particle at the top is not a proton but the antimatter version of an electron, called a *positron*.

The two intermediate layers show matter and antimatter versions of *quarks*. When combined in triplets, the matter versions of these particles form protons and neutrons and thus are extremely common. They come in two varieties, *up* (u , with $\frac{2}{3}$ electric charge) and *down* (d , with $\frac{1}{3}$ electric charge). Each layer has three particles because quarks hold an additional type of charge, color, that comes in three varieties called "red," "green," and "blue." While these charges have nothing to do with light, the reason for the labels is that, like red, green, and blue light that together form white light, color charges disappear when all three combine. Color charges also have antimatter equivalents called "cyan" (anti-red), "magenta" (anti-green), and "yellow" (anti-blue). The label on the bottom layer or corner of the cube is the Greek letter ν (ν), which is confusingly similar to a lowercase v . Physicists call this notoriously hard-to-capture particle that this letter represents the *neutrino*. This fermion is unique because it never stops moving, sailing constantly through space at close to lightspeed and rarely interacting with other forms of matter.

Since the layers of the Glashow cube alternate between antimatter and matter, some key particles, such as the down quark and electron, are technically missing, though represented by their antimatter equivalents. The reason for this absence is that there's a second cube that Glashow did not bother to show. Glashow drew only the *up* cube, which contains the *up* quark and particles with positive (or zero) electric charge.

Figure 5 shows the second or *down* Glashow cube. It contains both the down quark and the electron.

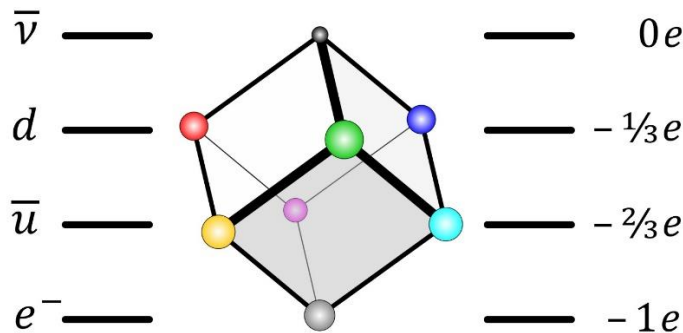


Figure 5. The down Glashow 3-cube.

Here's the odd thing: Since the *up* and *down* Glashow cubes share a common color structure and electrical charge direction in which up means more positive — one can view them as *copies* of each other despite containing different particles. As copies, they can be “pushed apart” to create a new cube with four dimensions. The “push” needed is adding or subtracting one electric charge unit to all particles in the cube. Figure 6 shows this two-way relationship.

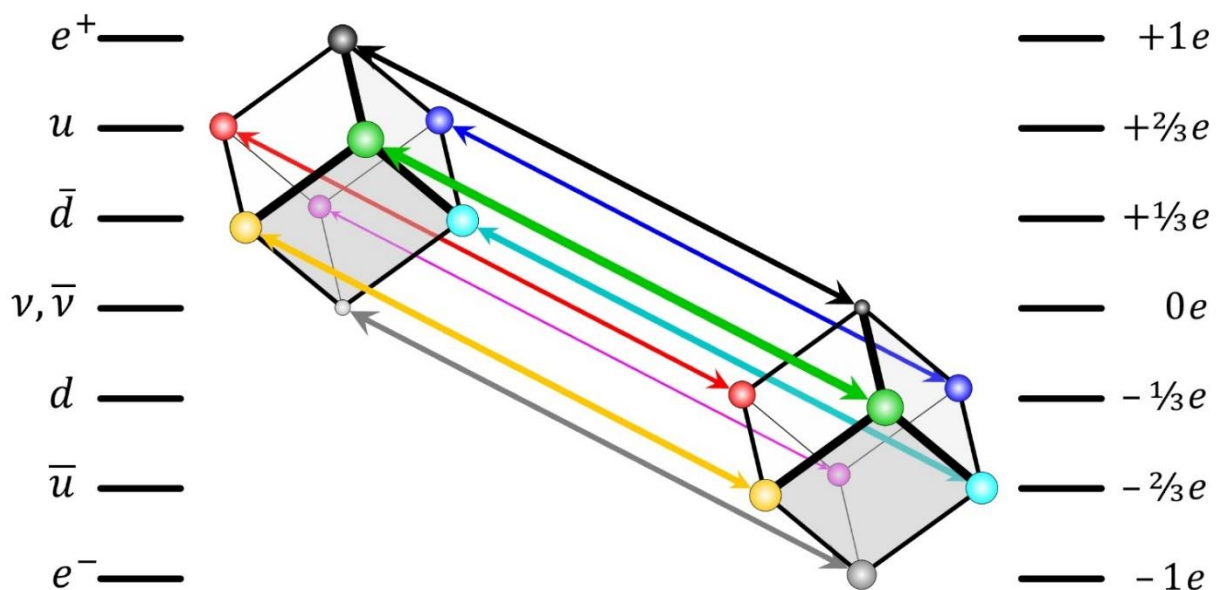


Figure 6. The up and down Glashow 3-cubes form a single 4-cube.

Fermions as Edges of 4-Cubes

Figure 6 should look familiar: It is the copy-and-push tesseract of Figure 2! The implication is that if you “push” particles in the bottom cube the right way (electric charge), you get particles in the top cube, and vice-versa.

Surely, this is just a geometric coincidence that has nothing to do with particle physics — right?

What might seem to be a mere topological coincidence ties in directly to one of the more interesting features of the Standard Model, the *weak interaction* or *weak force*. You’ve likely heard of the weak force as the mysterious — and yes, weak — force that makes fusions possible in the Sun by *occasionally* converting protons into easily fused neutrons. (An aside: It is fortunate that this force is so weak since otherwise, the Sun would have exploded the moment it formed.) Peering closer, what the weak force does in such proton-to-neutron conversions is, with the help of an electron, convert one *up* quark in the proton into a *down* quark. The quark flip drops the proton’s total charge by one unit, turning it into a neutron. In Figure 6, the two-way diagonal lines between the *up* and *down* cubes represent all interactions possible. The weak-force particle W^+ converts *down* fermions into their *up* partners, while another called W^- pushes *up* fermions into the *down* partners. The more precise physics name for these diagonal bridges is “the T_3 component of the weak isospin,” but we’ll call them *tesseract bridges*.

The intimate geometric connection of the *up* and *down* fermion groups also permits a different way of thinking about fermions, such as *tesseract bridge vectors*. Figure 7 shows the *up* vectors that correspond to *up* fermions and antifermions with a positive charge, plus the neutrino at the bottom of the *up* cube:

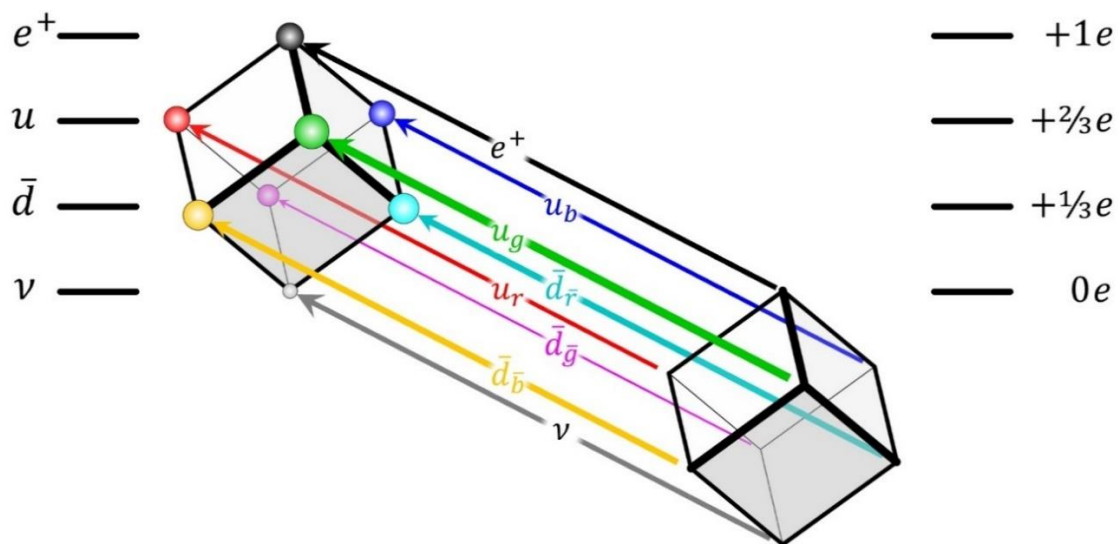


Figure 7. Representing *up* fermions as tesseract bridge vectors (versus points).

Figure 8 shows *down* bridge vectors for negative fermions and antifermions, plus the anti-neutrino.

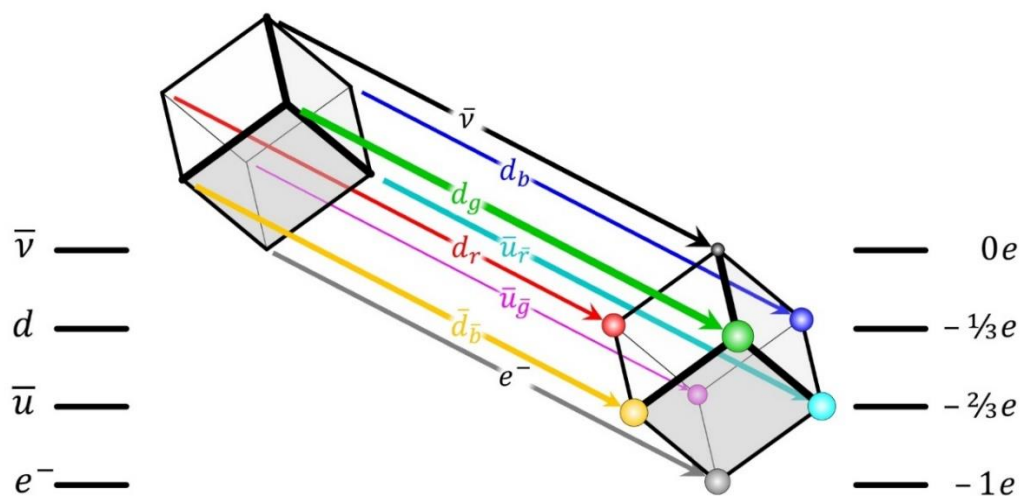


Figure 8. Representing “down” fermions as tesseract bridge vectors (versus point particles).

There is no critical need to interpret these fermions as vectors in a tesseract space since one can only look at the results of weak interactions and view those results as independent particles. However, when a simple geometric relationship persists without exception in the data, paying closer attention is a good idea. Just because a fermion looks like a point in three-dimensional space does not eliminate it from having an additional structure in some higher-dimensional space, with the three-dimensional point being only how it looks in a lower-dimensional space.

The Three Generations of Fermion Tesseracts

But what about the penteract? Why did I bother to show a still-higher dimensional cube? The reason is a still-outstanding problem of physics: the 4-cube structure of [Figure 6](#) repeats exactly three times in particle physics, with each repeat growing more massive. Physicists call these *generations*, and their existence indicates additional hidden geometric structure. [Figure 9](#) shows an example — likely wrong! — of mapping them into a penteract.

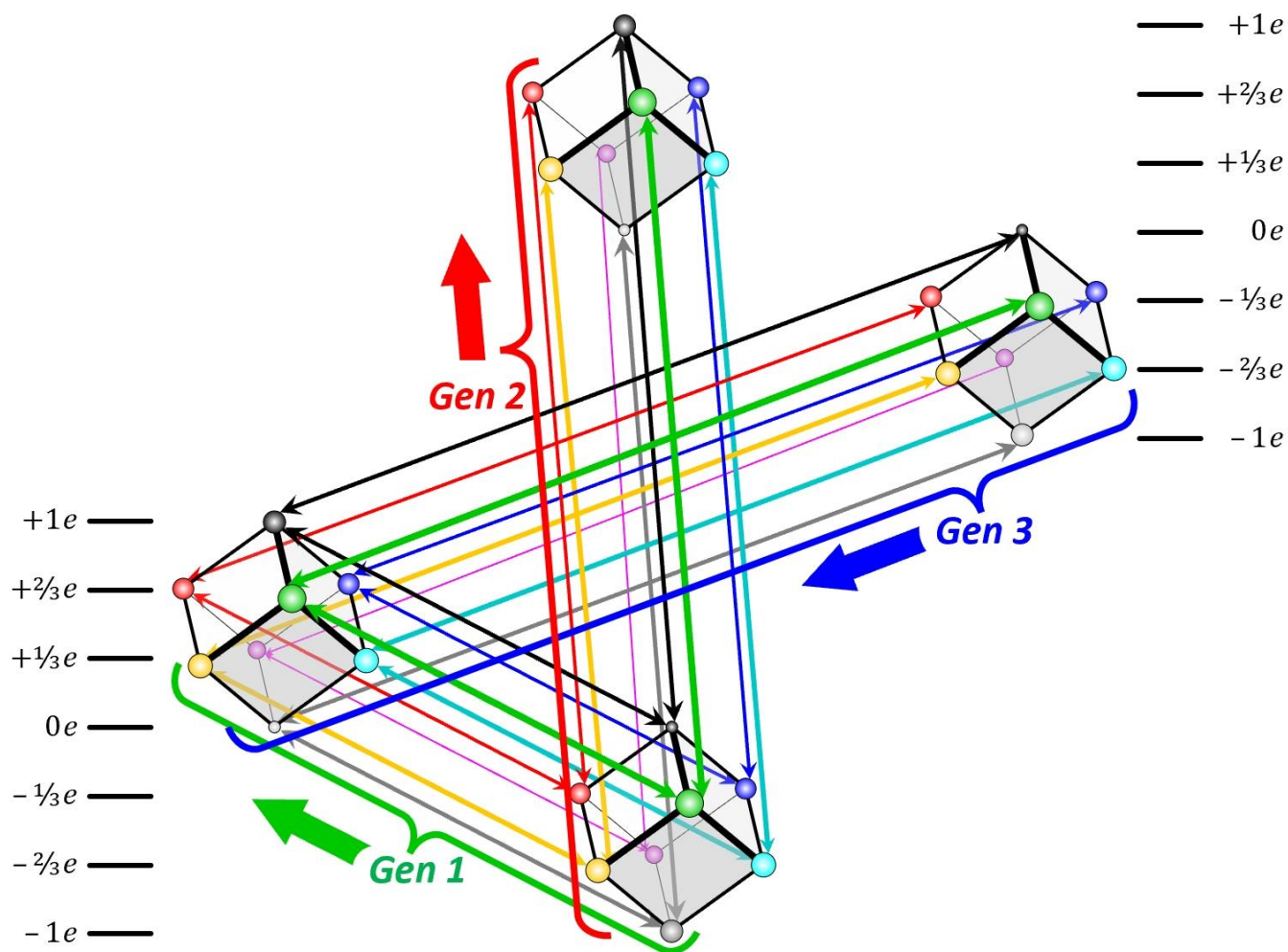


Figure 9. One possible example of representing all three fermion generations as penteract bridge vectors.

The Intransigence of Simplicity

The Standard Model of particle physics is necessarily vastly more complicated than any of these simple cube figures. For example, I have not even addressed the critical issue of chirality — mirror imaging of particles — or how chirality fits into these figures. The point here is the one I mentioned earlier: When simple geometry emerges consistently in experimental data, one should take it seriously *in its own right* without assuming that “only” the

vastly more complicated models for calculating precise results are the only way to gain further insights. The persistent simple forms may provide insights that are harder to discern in more computationally complete models.

What follows is a specific example of how intransigent simplicity might have much deeper meanings. [Figure 10](#) shows a slightly different way of organizing the *up* and *down* Glashow cubes. Instead of focusing on weak force (bridge) transitions, this arrangement focuses on the unique combinations of electric and color charges that identify and distinguish the 12 fermions of each generation. Rather than separating electric and color charges as fully unique, this representation emphasizes that while electric charges can occur without color charges — the positron and electron at the top and bottom of the charge stack — one *never* sees color occurring as a separate particle charge in experimental physics. Instead, color charges *occur* only in combination with either $\frac{1}{3}$ or $\frac{2}{3}$ of a unit of electric charge. One can easily miss noticing this if your focus is solely on a complicated set of equations that *assume* color occurs as a separate entity from electric charge, even though that never happens in nature. Conversely, if you represent charge using the stubbornly persistent Glashow cubes, the connection becomes obvious: Electric and color charges *always* form singular, well-defined “chromoelectric” charges (displacement vectors) in Glashow space.

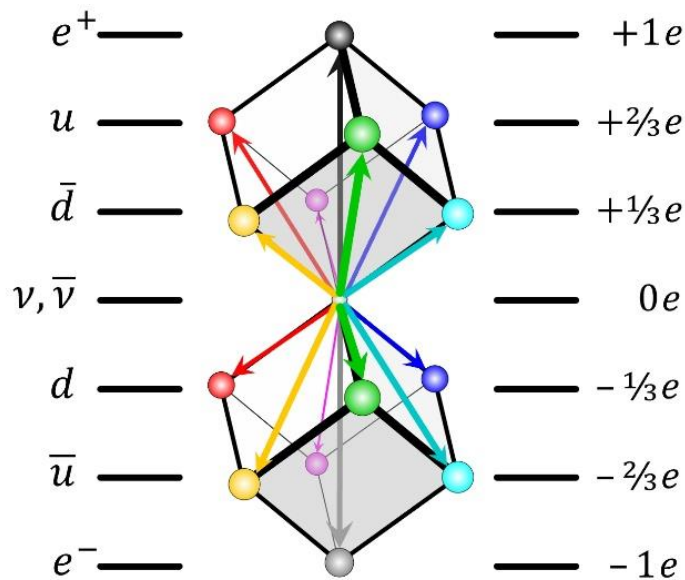


Figure 10. Unified fermion charges as vertical and non-vertical vectors.

The ease with which combined electric and color vectors emerge in Glashow charge construction raises an unsettling proposition: Is it possible that decades of attempts to unify the electric and color forces — called grand unified theories — failed because *the two charge types never separated*? The alternative is that each quark has a single charge vector that *looks* like a mix of electric and color charges only because we have a historical preference to interpret the data that way. Historically, curious minds discovered electric charge millennia before color charge first, so *of course*, we expect the electric charge to remain separate and fundamental at all levels of reality. That view, however, is a mathematical habit accompanied by a great deal of sunk-cost equation derivation using the full-separation model. The irony is that experimental physics results do *not* support the full separation of electric and color charges due to color charges never occurring in nature without a precisely defined accompanying ratio of electric charge. This intransigent pairing implies a much closer linkage between the two types of charge.

A Timely Speculation

Let's take that idea a bit further. If electric and color charges are the same entity, what does that say about the *angles* of the fermion charge vectors in [Figure 10](#)? This analysis does not need to get complicated, but instead can focus on the difference in behaviors of the four levels of fermions in each Glashow cube.

The simplest distinction of the charge types is this: Fermions with fully vertical charge vectors — electric-only or no charge — can roam indefinitely far in the universe. Conversely, fermions with off-vertical charge vectors — charges that include a color charge component — are *confined* to the tiny space inside particles such as protons or neutrons. The relativistic relationship of space and time permits one to restate that connection differently: Fermions with *vertical* charges can roam any distance in *time* since an indefinite duration is required to roam in space at light or sublight speeds. Conversely, off-vertical charges confine fermion in space and limit their local ability to experience historical, information-producing time. Any attempt to “sail off” on a trajectory long enough to produce an irreversible record gets redirected and confined to the diameters of particles such as protons and neutrons.

Physicists have written volumes about the confinement process of quarks. However, the guiding principle here is that there is geometric simplicity in these processes, a simplicity that endures and shows through no matter how complex the full process becomes. The hypothesis thus becomes this: Each charge angle is associated with its definition of *time*, but not all such times are created equal. Only the vertical charge direction — the electric charge or Maxwell displacement axis — is compatible with long-range, historical, information-rich classical time. Fermions with off-vertical charges attempt what amounts to limited local definitions of time that cannot endure beyond the scale of a proton or neutron. Only when the three colors cooperate and wind around each other in a tight dance can they exhibit the simple electric charge of an electron that allows large motions through space and time.

Figure 11 identifies the 36 quarks and antiquarks that, under this charge-time hypothesis, cannot endure in large-scale spacetime. Interestingly, this is the same number of zero-dimensional scalar fields proposed by Latham Boyle and Neil Turok [2] to prevent quantum field theory from collapsing the vacuum. While derived from very different perspectives, these off-axis time and vacuum cancelation numbers may be two ways of looking at the same issue.

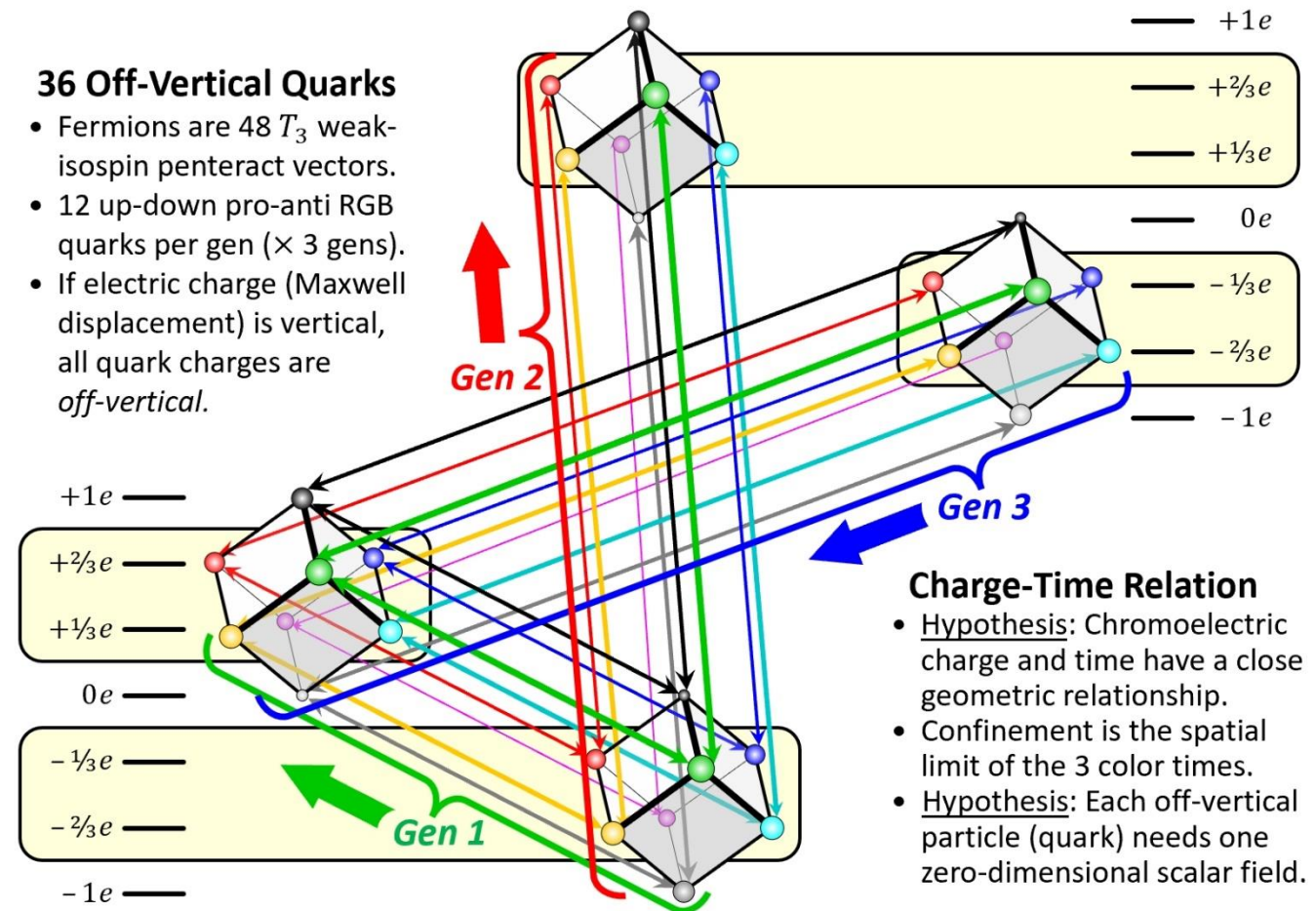


Figure 11. Non-vertical-penteract fermions and the 36 Boyle-Turok zero-dimensional scalars.

Summary

One must never underestimate the power of continuum (real-number) physics maths, especially when calculating precise answers. However, a balancing principle is this: Never let the potentially infinite complexity of continuum representations of tiny systems take away from the possibility that stubbornly persistent patterns hiding beneath that complexity can provide new perspectives on issues not obvious in the more detailed representations. The fact that cubes show up so vividly and in so many ways in the construction of the Standard Model fermions is vanishingly unlikely to be a coincidence but instead informs us that much of that seeming complexity may instead be *generated* by these simpler forms as they interact in large ensembles. Occasionally, these very different approaches of deep exploration of continuum forms and elaborations of extremely simple geometric relations that keep showing up in particle data can arrive at similar conclusions, such as the number of zero-dimensional fields postulated in the Boyd-Turok dual universe model. Such surprises merit investigation, even if they cross steep conceptual boundaries.

References

- [1] S. L. Glashow, *The Future of Elementary Particle Physics* [HUTP-79/A059], Harvard University Preprints, Jul. (1979). <https://inspirehep.net/literature/144466>. The original hand-drawn Glashow cube is on page 29 as the first figure in *Section III, Let the Desert Bloom!*
- [2] L. Boyle and N. Turok, *Cancelling the vacuum energy and Weyl anomaly in the standard model with dimension-zero scalar fields*, arXiv preprint arXiv:2110.06258 (2021). <https://arxiv.org/abs/2110.06258>